ANALYSIS OF MULTIPORT WAVEGUIDE STRUCTURES BY A HIGHER-ORDER FDTD METHODOLOGY BASED ON NONORTHOGONAL CURVILINEAR GRIDS

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ABSTRACT

A generalized methodology for the construction of nonstandard higher-order finite-difference time-domain schemes, as well as their application to complex electromagnetic problems in curvilinear non-orthogonal coordinate systems, are presented in this paper. As a consequence, a new class of low-dispersion operators is designed for the approximation of spatial and temporal derivatives. Their extension to curvilinear non-orthogonal coordinates is attained by a higher-order variation of the covariant and contravariant vector component theory, in which all metric terms are taken into account. Finally, the proposed method is validated by the analysis of diverse multiport microwave structures with realistic features.

INTRODUCTION

The vast technological growth observed during the past decades in microwave industry, continuously increases the requirements for analysis and design of complicated waveguide systems, such as couplers, filters, phase shifters and generally multiport structures. Due to their great importance, these components have been an issue of intensive scientific research by advanced numerical techniques (MoM [1], FEM [2], FDTD method [3], mode matching method [4]-[5] etc.). Unfortunately, they all have severe shortcomings that limit their applicability. For instance, the discretization of circular cross section waveguides or arbitrarily inclined slots by means of Yee's algorithm, which, appears to be the most popular of the numerical methods, produces low accuracy results and significant dissipation and dispersion errors [3]. On the other hand, the traditional ways of excitation necessitate the elongation of uniform parts. These deficiencies may be overcome via highly refined meshes or generalized unstructured lattices [6], which however, have prohibitively excessive demands in CPU time and memory capacity.

It is the purpose of this paper to introduce a new nonorthogonal higher-order FDTD methodology, based on curvilinear nonstandard schemes, for the feasible and efficient treatment of complicated 3-D waveguide structures. A class of higher-order dispersionless operators for the exact approximation of spatial and temporal derivatives is constructed in a general curvilinear system. The basic premise of the algorithm lies on the representation of electromagnetic fields by means of a modified higher-order covariant and contravariant strategy, which stems from an efficient solution of the div-curl problem. Apart from the leapfrog time integration process, the general Runge-Kutta operators are alternatively utilized, whereas a new self-adaptive compact central difference procedure treats the inevitably widened spatial stencils. Consequently, reduced spurious dispersion, anisotropy and lattice reflection errors are obtained. The proposed methodology was applied to the analysis of various multiport waveguide junctions incorporating discontinuities (slots and irises).

THE NONSTANDARD HIGHER-ORDER FDTD SCHEME

The innovative methodology is essentially based on the representation of the spatial and temporal derivatives by means of the accurate nonstandard higher-order (HO) differencing procedure. The proposed approximators are, respectively, given by

$$\mathbf{S}_u[f] = c_1 \mathbf{D}^u_{\delta u}[f] + c_2 \left(\mathbf{D}^u_{3\delta u}[f] + \frac{f\big|^t_{u-\frac{1}{2}\delta u} - f\big|^t_{u-\frac{3}{2}\delta u}}{\delta u} \right), \quad (1)$$

$$\mathbf{T}_u[f] = \left(f\big|_u^{t+\delta t/2} - f\big|_u^{t-\delta t/2} \right) / \, m_\omega(\delta t) - (\delta t^2/24)\partial_{ttt} f\big|_u^t, \quad (2)$$

where u belongs to a general coordinate system (u,v,w), coefficients c_i are chosen to eliminate the appearance of any anisotropy deficiencies and $\mathbf{D}_\Delta$ $(\Delta=\delta u, 3\delta u)$ is the three-dimensional nonstandard operator, defined as

$$\mathbf{D}^u_\Delta[f] \equiv \frac{1}{m_k(k\Delta)} \left(q_1 \mathbf{d}^{(1)}_{u,\Delta}[f] + q_2 \mathbf{d}^{(2)}_{u,\Delta}\left[f^t_u\right] + q_3 \mathbf{d}^{(3)}_{u,\Delta}[f] \right). \quad (3)$$

The $m_k(k\Delta)$ and $m_\omega(\delta t)$ are correction functions (of sinusoidal form) selected to minimize the inevitable error generated by the derivative approximations in (1) and (2), respectively, and also to significantly enhance the tech-

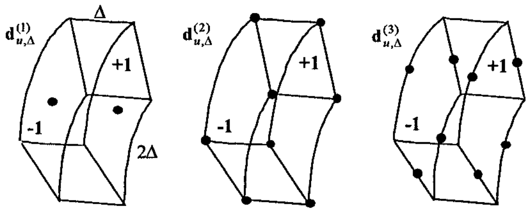

Fig. 1. Graphical representation of the curvilinear finite difference operators. The numbers at the faces indicate the sign of summation in (4).

nique's dispersion and dissipation features. The difference operators of (3), $\mathbf{d}^{(p)}$ for $p = 1,2,3$, are described according to the geometry of the elementary cell (Fig. 1) by the following expressions

$$\mathbf{d}^{(1)}_{u,\Delta}[f] = f\big|^t_{\Delta/2,0,0} - f\big|^t_{-\Delta/2,0,0}, \tag{4a}$$

$$\mathbf{d}^{(2)}_{u,\Delta}[f] = \frac{1}{2}\begin{pmatrix} f\big|^t_{\Delta/2,\Delta,\Delta} + f\big|^t_{\Delta/2,\Delta,-\Delta} + f\big|^t_{\Delta/2,-\Delta,\Delta} \\ + f\big|^t_{\Delta/2,-\Delta,-\Delta} - f\big|^t_{-\Delta/2,\Delta,\Delta} - f\big|^t_{-\Delta/2,\Delta,-\Delta} \\ - f\big|^t_{-\Delta/2,-\Delta,\Delta} - f\big|^t_{-\Delta/2,-\Delta,-\Delta} \end{pmatrix}, \tag{4b}$$

$$\mathbf{d}^{(3)}_{u,\Delta}[f] = \frac{1}{4}\begin{pmatrix} f\big|^t_{\Delta/2,\Delta,0} + f\big|^t_{\Delta/2,-\Delta,0} + f\big|^t_{\Delta/2,0,\Delta} \\ + f\big|^t_{\Delta/2,0,-\Delta} - f\big|^t_{-\Delta/2,\Delta,0} - f\big|^t_{-\Delta/2,-\Delta,0} \\ - f\big|^t_{-\Delta/2,0,\Delta} - f\big|^t_{-\Delta/2,0,-\Delta} \end{pmatrix}. \tag{4c}$$

For brevity, in (4) only the respective lattice space increments towards the u,v,w directions are indicated (i.e. the notation $-\Delta/2,0,\Delta$ means $u-\Delta/2,v,w+\Delta$). The crucial q parameters of (3) ensure algorithmic stability and structural well-posedness and are calculated via

$$q_1 = p + s(1-p)/3\ , \quad q_2 = s(1-p)/3\ ,$$

$$q_3 = 1 - p - 2s(1-p)/3\ ,$$

with

$$p(k) = \frac{\cos k_u \cos k_v - \cos k}{1 + \cos k_u \cos k_v - \cos k_u - \cos k_v}, \tag{5a}$$

$$s(k) = \frac{pR_A + (1-p)R_B - (\cos k - 1)}{(p-1)(R_A + R_B - 2R_C)}, \tag{5b}$$

while the coefficients R_A, R_B and R_C are mathematically expressed in terms of the wave number components, as

$$R_A = \cos k_u + \cos k_v + \cos k_w - 3\ , \tag{6a}$$

$$R_B = \cos k_u \cos k_v \cos k_w - 1\ , \tag{6b}$$

$$R_C = \tfrac{1}{2}(\cos k_u \cos k_v + \cos k_u \cos k_w + \cos k_v \cos k_w - 3)\ . \tag{6c}$$

Another attribute of the nonstandard HO FDTD schemes, that has to be dealt with, is the widened spatial stencil near perfectly conducting interfaces and absorbing walls. This difficulty is circumvented by a general class of self-adaptive compact operators which guarantees the thorough modeling of complex applications. In particular, they can be expressed (in central or nonsymmetric version) by the Hermite formula

$$\sum_{p=-1}^{1}\left(a_p f_{i+p} + b_p f'_{i+p} + c_p f''_{i+p}\right) = 0 \tag{7}$$

where a_p, b_p, c_p are unknown calculable real coefficients.

Finally, the new discretization methodology is effectively combined with the fourth-stage Runge-Kutta integration scheme (except of the leapfrog one), which staggers the variables in space but not in time. Generally, it is given by

$$f\big|^{n+1}_{i,j,k} = \sum_{m=1}^{M} \frac{(-\delta t \Xi)^m}{m!} \cdot f\big|^n_{i,j,k}, \tag{8}$$

in which Ξ is the spatial discretization matrix and M the order of the integrator. Being conditionally stable and unaffected by the presence of compact operators as its second-order counterpart, this integrator has proven to be 1.4 times more efficient than the fourth-order leapfrog one.

HO OPERATORS IN CURVILINEAR GRIDS

The adjustment of the aforementioned differencing scheme to a general curvilinear coordinate system is not straightforward, since the basis on which the field quantities are expressed, affects the consistency of the numerical solution of Maxwell's equations. An improper selection may give rise to Cristoffel symbols, which cannot be accurately computed [7].

In order to overcome this strenuous div-curl problem, we develop a new algorithm incorporating a fully conservative HO rendition of the covariant/contravariant theory, which takes all metric terms into account. For this purpose, a Helmholtz-type decomposition, which computes the desired electric or magnetic vector via the projection of the curl onto the space of divergence-free vectors, is used [8].

Assuming a general nonorthogonal right-handed curvilinear coordinate system (u,v,w) and that this mapping is smooth enough, any vector $\mathbf{F}$ can be decomposed with respect to the contravariant $\mathbf{a}^1,\mathbf{a}^2,\mathbf{a}^3$ or the covariant $\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3$ base system as

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$$\mathbf{F}=\sum_{i=1}^{3}(\mathbf{a}_i\cdot\mathbf{F})\mathbf{a}^i=\sum_{i=1}^{3}f_i\mathbf{a}^i=\sum_{i=1}^{3}(\mathbf{a}^i\cdot\mathbf{F})\mathbf{a}_i=\sum_{i=1}^{3}f^i\mathbf{a}_i\,. \quad (9)$$

The quantities f^i and f_i denote respectively the contravariant and covariant components of **F** which, due to their reciprocity, satisfy the relation $\mathbf{a}_i\cdot\mathbf{a}^j=\delta_{ij}$ (δ_{ij} is the Kronecker's delta).

Derivative approximation described in such a generalized coordinate system, yields an accurate HO curvilinear FDTD curl operator, which is subsequently applied to Maxwell's equations. Hence, the general matrix form of Ampere's and Faraday's law becomes

$$\left(\mathbf{I}+\tfrac{1}{2}\mathbf{A}_t\right)\mathbf{E}_c^{n+1}=\left(\mathbf{I}-\tfrac{1}{2}\mathbf{A}_t\right)\mathbf{E}_c^{n}+\mathbf{G}^H\mathbf{S}\left[\mathbf{H}_c^{n+1/2}\right] \quad (10)$$

$$\left(\mathbf{I}+\tfrac{1}{2}\mathbf{B}_t\right)\mathbf{H}_c^{n+1/2}=\left(\mathbf{I}-\tfrac{1}{2}\mathbf{B}_t\right)\mathbf{H}_c^{n-1/2}+\mathbf{G}^E\mathbf{S}\left[\mathbf{E}_c^{n}\right] \quad (11)$$

where c denotes the covariant electric components and $\mathbf{A}_t$, $\mathbf{B}_t$, and $\mathbf{G}^H$, $\mathbf{G}^E$, derivative and metric tensor operators, respectively. Therefore, the curvilinear div-curl problem, which is reduced to the computation of a vector field **F** assuming that its curl and its divergence are known, can now be efficiently treated by our algorithm.

Finally, the stability criterion of our method becomes

$$c\delta t\le\frac{3\sin^{-1}(0.7)}{\pi\left(\sum_{l=1}^{3}\sum_{m=1}^{3}\frac{g^{lm}}{\delta\zeta^l\delta\zeta^m}\right)^{1/2}},\ \text{ with } \zeta^{l,m}=u,v,w\,. \quad (12)$$

ANALYSIS OF MULTIPORT STRUCTURES

The presented generalized nonstandard higher-order FDTD schemes were implemented for the analysis of diverse three-dimensional realistic multiport waveguide structures. For this purpose, a pulsed modulated excitation, which imposes the source plane several cells away from the ABC's plane in order to separate incident and reflected fields, was used [9]. Considering a distance of d_c cells from the boundary, E_v is expressed as

$$E_{v+d_c}^{n+1}=FDTD\,update+L(u,v)\sum_{s=1}^{N}\sin(2\pi f_s t-\beta_s w)$$

where $L(u,v)$ represents the pulse's spatial profile. Hence, no artificial elongation of uniform parts is needed and an initial significant reduction in the computational resources is obtained. It is mentioned herein that the structures are terminated by higher-order versions of existing PMLs [10]-[11], which exhibit much better reflection and dispersion properties, compared to the conventional ones.

The first application was a T-junction consisting of orthogonal waveguides, where the connection between the feeding and the coupling parts is obtained through an inclined slot of angle θ with respect to the coupling part's axis. The waveguides' dimensions are $22.86\times10.16\text{mm}^2$ whereas the slot's width and thickness are 16mm and 0.8mm, respectively. Fig. 2 depicts the computed coupling coefficient, C, between ports 1 and 3, which is defined as $20\log|S_{31}|$, with S_{31} being the corresponding S-parameter. The efficiency of our algorithm, as compared to the reference Method of Moment solution of [1], is very promising. Furthermore, the improved accuracy of the proposed higher-order FDTD method, with a simultaneous significant reduction of the computational grid, is illustrated in Fig. 3.

The magnitude of various S-parameters of a six-port cross-junction (each port is orthogonal with dimensions $15.799\times7.899\text{mm}^2$) is illustrated in Fig. 4 and compared with the results of [4]. Again the mesh reduction and therefore, the overall gain in computational time, are remarkable. Finally, the variation of S_{21}-parameter for a curvilinear iris-coupled resonator, is analyzed. Fig. 5 demonstrates the promising accuracy and memory savings (almost 80% of Yee scheme) achieved by the HO FDTD scheme.

CONCLUSION

In this paper, we have mathematically established and systematically derived a class of dispersionless nonstandard higher-order FDTD operators, in general non-orthogonal curvilinear coordinate systems. The div-curl problem that such an approach involves, was overcome by means of a higher-order covariant and contravariant theory that considers all metric terms. The proposed method has proven

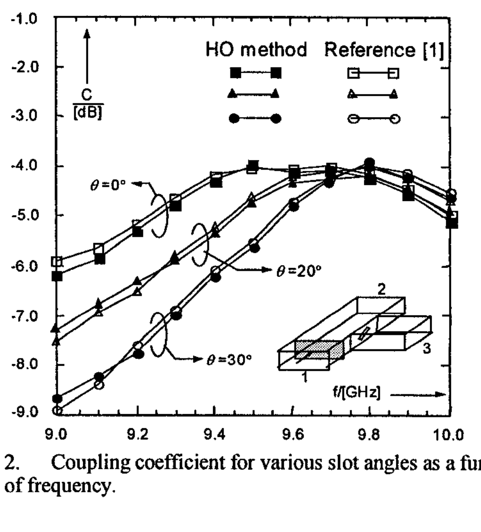

Fig. 2. Coupling coefficient for various slot angles as a function of frequency.

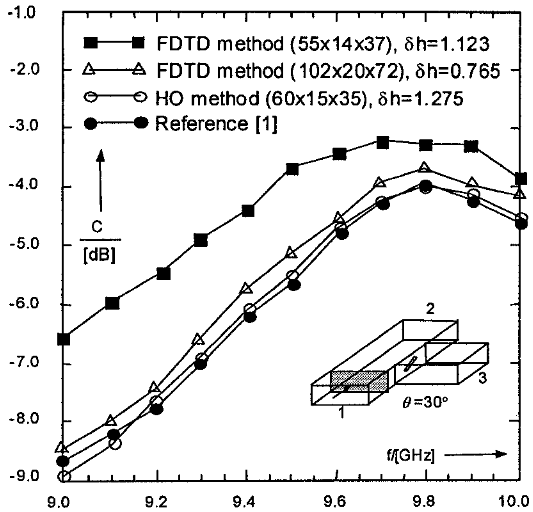

Fig. 3. Coupling coefficient for slot angle $\theta = 30°$ computed by the FDTD, the HO FDTD and the MoM methods.

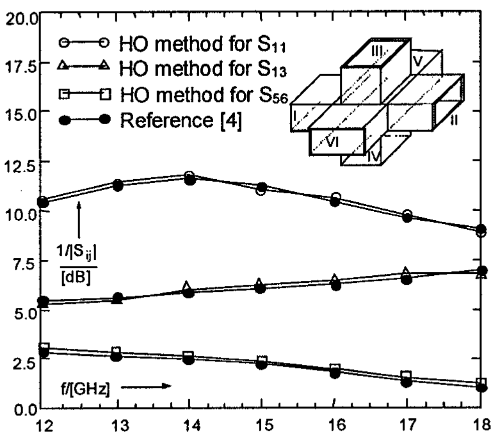

Fig. 4. S-parameter magnitude for a six port junction.

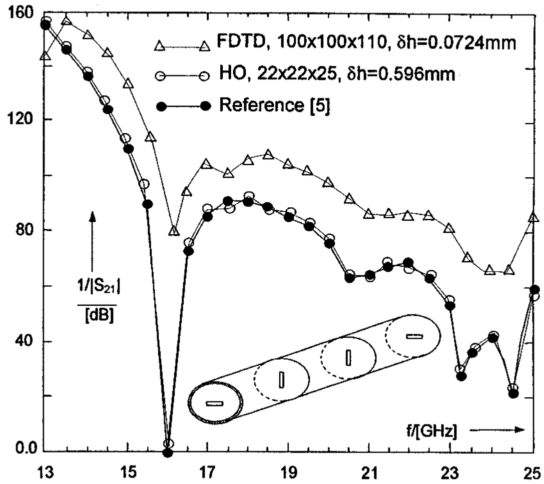

Fig. 5. The S_{21}-parameter magnitude of a cylindrical iris-coupled resonator.

to be remarkably precise and computationally inexpensive when implemented to the analysis of a variety of multiport waveguide structures, incorporating modern technological features such as inclined slots.

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